Acquisition of observer mapping in driftless, non-holonomic system

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In this study we develop an exploratory method for non-holonomic driftless systems with unknown state equation and sensor mapping to enable controllability. For that purpose, we acquire an approximated mapping from the sensor space to a virtual linear state space by the study of the Jacobian of the sensors, therefore circumventing the lack of knowledge of the underlying state equation of the system and mapping of the sensors.

Key Words: Mobile robot, self-localization, mapping, non-holonomic

1 Introduction

In the study of non-holonomic dynamic systems such as the unicycle, the sensor model is commonly a given and hardcoded into the algorithms[1][2]. Here we explore the reconstruction of unknown sensor mappings to enable control in the context of driftless, non-holonomic systems.

The goal of this work is to recover the sensor mappings of a robotic system by applying specific motor inputs and observing the changes in the sensor outputs with the intention of building a dataset to reconstruct the function that relates the robot state variables with the sensor signal.

We define three vector spaces that correspond to the true state space $\mathcal{X}$ of the non-holonomic robot, the sensor space $\mathcal{S}$ that maps non-linearly to $\mathcal{X}$ and a virtual state space $\mathcal{V}$. The sensor configuration defines an unknown isomorphic mapping $\phi : \mathcal{X} \longrightarrow \mathcal{S}$, whose reconstruction is the object of this research. For that purpose, we define the inverse mapping $\hat{\phi}^{-1} : \mathcal{S} \longrightarrow \mathcal{V}$ between the sensor space and the virtual space and design $\hat{\phi}^{-1}$ assuming that the original state space $\mathcal{X}$ is in chained form [3][4].

2 Methods

In order to obtain, $\hat{\phi}^{-1}$, we designed a method to explore the sensor space in an efficient manner and thus map the sensor readings to the expected state equation in chained form. The first step is to find the optimal directions to explore the space, followed by data acquisition and then function approximation.

2.1 Initial Orthogonal Direction (IOD)

The subspace that is not immediately controllable with the system inputs from the state at $t = 0$ is what we call the *Initial Orthogonal Direction* and it forms the subspace $\mathcal{J}_{IOD}$, which is orthogonal to the subspace generated by the non-IOD directions $\mathcal{J}_0$. The Jacobian indicates which directions of the sensor space are immediately controllable with the system inputs and their linear combination form the subspace $\mathcal{J}_0$. Since the system is non-holonomic, the rank of the Jacobian is smaller than the rank of the state space.

We start by identifying the IOD because we have no prior information about ϕ. The Jacobian of the sensor readings at state $\boldsymbol{s}_i$, where i indicates the sample index, with respect to the control input $\boldsymbol{u}$ is $J_i = \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{u}}\big|_{\boldsymbol{s}=\boldsymbol{s}_i}$. The state equation in sensor space is given by

$$\dot{\boldsymbol{s}}_i = \boldsymbol{g}(\boldsymbol{s}_i)\,\boldsymbol{u} = \boldsymbol{g}_1(\boldsymbol{s}_i)\,u_1 + \boldsymbol{g}_2(\boldsymbol{s}_i)\,u_2. \quad (1)$$

If we apply control input $\boldsymbol{u} = \begin{bmatrix} u_1 & 0 \end{bmatrix}^\intercal$, then $\dot{\boldsymbol{s}}_{i,1} = \boldsymbol{g}_1(\boldsymbol{s}_{i,1})u_1$ so

$$\frac{\partial \dot{\boldsymbol{s}}_{i,1}}{\partial u_1} = \frac{\partial \boldsymbol{g}_1(\boldsymbol{s}_{i,1})\,u_1}{\partial u_1} + \frac{\partial \boldsymbol{g}_2(\boldsymbol{s}_{i,1})\,u_2}{\partial u_1} = \boldsymbol{g}_1(\boldsymbol{s}_{i,1}) \approx \frac{\Delta \boldsymbol{s}_{i,1}}{u_1\,\Delta t} \quad (2)$$

Similarly for $\boldsymbol{u} = \begin{bmatrix} 0 & u_2 \end{bmatrix}^\intercal$ and substituting, we arrive at

$$J_i = \begin{bmatrix} \frac{\partial \dot{\boldsymbol{s}}_{i,1}}{\partial u_1} & \frac{\partial \dot{\boldsymbol{s}}_{i,2}}{\partial u_2} \end{bmatrix} \approx \begin{bmatrix} \frac{\Delta \boldsymbol{s}_{i,1}}{u_1\,\Delta t} & \frac{\Delta \boldsymbol{s}_{i,2}}{u_2\,\Delta t} \end{bmatrix} \quad (3)$$

Therefore, by applying a constant input for a fixed amount of time and measuring the variations in the sensor readings, we can obtain the Jacobian of the sensor readings. The inputs are activated in sequential patterns to find the state where the Jacobian components are biggest in the IOD (Fig. 1).

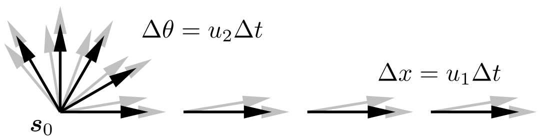

Fig.1: The Jacobian of the system at any state is obtained by applying small inputs, sampling the sensor readings and returning to the initial position, sequentially for each input at several states as indicated. The Jacobian is used to obtain the *Initial Orthogonal Direction* (IOD).

We now obtain analytically the orthogonal components of every $\boldsymbol{p} := \frac{\Delta \boldsymbol{s}}{u\,\Delta t}$ of $\mathcal{J}_0$ (we omit index i in $\boldsymbol{s}$) to find the closest configuration to the ideal IOD and the required input to traverse the IOD. Let $B_0 = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_m)$ be an orthogonal basis for $\mathcal{J}_0 = (\boldsymbol{p}_1, \ldots, \boldsymbol{p}_m)$, with m the number of control inputs, obtained by linear combination of the vector components of J_0. We start by decomposing $\boldsymbol{p}$ into two vectors $\boldsymbol{q} \in \mathcal{J}_0$ and $\boldsymbol{r} \in \mathcal{J}_0^\perp$, denoted the projection and the orthogonal respectively, such that $\boldsymbol{p} = \boldsymbol{q} + \boldsymbol{r}$. But any vector $\boldsymbol{q} \in \mathcal{J}_0$ is a linear combination of $(\boldsymbol{b}_1, \ldots, \boldsymbol{b}_m)$:

$$\boldsymbol{q} = \sum_{k=1}^{m} \lambda_k\,\boldsymbol{b}_k, \quad (4)$$

where

$$\lambda_k = \frac{\langle \boldsymbol{b}_k, \boldsymbol{q} \rangle}{\langle \boldsymbol{b}_k, \boldsymbol{b}_k \rangle} \quad (5)$$

and $\langle \boldsymbol{b}_k, \boldsymbol{q} \rangle$ is the inner product between $\boldsymbol{b}_k$ and $\boldsymbol{q}$. Then,

$$\boldsymbol{q} = \sum_{k=1}^{m} \frac{\langle \boldsymbol{b}_k, \boldsymbol{q} \rangle}{\langle \boldsymbol{b}_k, \boldsymbol{b}_k \rangle}\,\boldsymbol{b}_k = \boldsymbol{p} - \boldsymbol{r}. \quad (6)$$

By the definition of orthogonal subspace we know that $\langle \boldsymbol{b}_k, \boldsymbol{r} \rangle = 0$ for all $k = 1, \ldots, m$, hence

$$\langle \boldsymbol{b}_k, \boldsymbol{q} \rangle = \langle \boldsymbol{b}_k, \boldsymbol{q} + \boldsymbol{r} \rangle = \langle \boldsymbol{b}_k, \boldsymbol{p} \rangle, \quad (7)$$

so

$$\boldsymbol{r} = \boldsymbol{p} - \sum_{k=1}^{m} \frac{\langle \boldsymbol{b}_k, \boldsymbol{p} \rangle}{\langle \boldsymbol{b}_k, \boldsymbol{b}_k \rangle}\,\boldsymbol{b}_k. \quad (8)$$

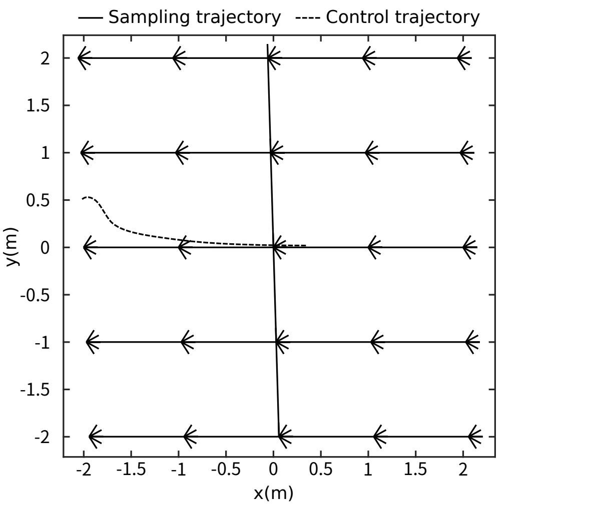

Fig.2: The trajectory in the true state space $\mathcal{X}$ of (a) sampling the sensor space to construct the dataset by supervised learning of the inverse mapping function $\hat{\phi}^{-1} : \mathcal{S} \longrightarrow \mathcal{V}$ and (b) linear control results in time-state space from $(x, y, \theta) = (-2, 0.5, \pi/4)$ to the origin.

Thus, $\boldsymbol{r}$ is the orthogonal component of $\boldsymbol{p}$ with respect to the subspace $\mathcal{J}_0$. (8) may be expressed in matrix form and extended to several vectors. We write the base B_0 of $\mathcal{J}_0$ as

$$B_0 = \left[\ \boldsymbol{b}_1 \quad \cdots \quad \boldsymbol{b}_m\ \right], \tag{9}$$

and $\boldsymbol{r}$ can be represented by

$$\boldsymbol{r} = \boldsymbol{p} - B_0\,(B_0^\intercal\, B_0)^{-1}\, B_0^\intercal\, \boldsymbol{p}. \tag{10}$$

Extending this result to the Jacobian $J_i = \begin{bmatrix}\boldsymbol{p}_{i,1} & \cdots & \boldsymbol{p}_{i,m}\end{bmatrix}$ and $R_i = \begin{bmatrix}\boldsymbol{r}_{i,1} & \cdots & \boldsymbol{r}_{i,m}\end{bmatrix}$, we obtain

$$R_i = (I - B_0\, B_0^\intercal)\, J_i, \tag{11}$$

given that B_0 is orthonormal. (11) is similar to the solution of the linear least square methods since it also involves projecting points to a target subspace.

The IOD corresponds to the greatest $\boldsymbol{r}_{i,j}$ of all R_i.

2.2 Data acquisition

The data acquisition stage involves sampling the sensor space at predefined points while simultaneously recording the trajectory history of the system assuming that the state equation is in chained form. We start by defining the origin of the system in $\mathcal{V}$ as the position of the system at the initial state. By traversing the state space as conveyed in Fig. 2, we can determine the expected state of the system inchained form and pair it with the sampled sensor readings. The IOD closest to the origin will be the only route used to traverse along the IOD in order to minimize the effect of non-planar topologies of the state equation of the system.

2.3 Function approximation

We applied a supervised learning method for approximating $\hat{\phi}^{-1}$ based on linear combination of Gaussian Radial Basis Functions[5]. The basis were chosen to form a $5 \times 5 \times 5$ grid fitted to the extreme values of the sensor readings. Variance was $\sigma^2 = 1.5^2$ multiplied by the distance between bases, and regularization coefficient was $\lambda = 0.5$

3 Simulation

We tested the approach on a simulated unicycle to assess the effectiveness of the method on a virtual non-holonomic system. We checked that the inverse mapping $\hat{\phi}^{-1}$ obtained as explained above by setting the starting position of the unicycle at any arbitrary position in the region covered by the exploration stage. We controlled the system by time-state control form[6] to the origin and logged the results. We tested several formulations for ϕ, from which here are three examples:

$$\phi_1(\boldsymbol{x}) = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}\ ;\ \phi_2(\boldsymbol{x}) = \begin{pmatrix} x+y \\ x-y \\ x\theta \end{pmatrix}\ ;\ \phi_3(\boldsymbol{x}) = \begin{pmatrix} e^x \\ \sinh(y) \\ \mathrm{atan}(\theta) \end{pmatrix} \tag{12}$$

Fig. 2 shows the resultant trajectory of the unicycle for ϕ_3. The unicycle found the optimum IOD to be slightly not perpendicular to the IOD in true state space because of the transformation $\phi(\boldsymbol{x})$ induced by the sensors. The controlled trajectory from $(-2, 0.5, \pi/4)$ to $(0, 0, 0)$ presents some distortion with respect to the stereotypical trajectory of a state-space controller because the values of $\hat{\phi}^{-1}$ between the basis points of the approximation are interpolations, thus the distortion of the trajectory reflects deviations of the interpolation with respect to the true inverse function ϕ^{-1}. Nevertheless, our approach was able to successfully control the unicycle system for the three examples of ϕ given in (12).

4 Conclusion

We proposed a method to reconstruct the observation mapping of a non-holonomic system by sampling the sensor space at regular intervals in a predefined pattern that maps the sensor inputs to an ideal representation of the state space. The proposed approach can handle a wide variety of sensor mappings. There are many applications that these results can benefit from. For example, autonomous vacuum cleaners may automatically infer their location from a randomly placed camera, and the information given by misplaced sensors in mobile robots can be automatically remodeled.

There are several limitations to this approach. The controllable region is limited to the region that has been explored during the data acquisition stage. Moreover, the described approach suffers from the curse of dimensionality in that a grid-like pattern of points in the sensor space must be sampled such that the number of points increment exponentially with the number of problem space dimensions. More sample points are needed in those regions where the sensor values vary quicker.

Another limitation is that when ϕ is not isomorphic as assumed, a unique mapping between vector spaces $\mathcal{S}$ and $\mathcal{V}$ may not be determined, leading to apparently random control laws in surjective sensor mappings. This limitation can be overcome by combining the information of redundant sensors, which will be our future work.

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