Three Dimensional Measurement of Object's Surface in Water Using the Light Stripe Projection Method

Atsushi Yamashita*, Hirokazu Higuchi*, Toru Kaneko* and Yoshimasa Kawata*
* *Department of Mechanical Engineering*
Shizuoka University
3–5–1 Johoku, Hamamatsu-shi, Shizuoka 432–8561, Japan
yamashita@ieee.org

***Abstract*— In this paper, we propose a three-dimensional (3-D) measurement method of objects' shapes in liquid by using the light stripe projection method. Usually, 3-D measurement by robot vision techniques is executed under the assumptions that cameras and objects are in aerial environments. However, an image distortion occurs when vision sensors measure objects in liquid. It is caused by the refraction of the light on the boundary between the air and the liquid, and the distorted image brings errors in a triangulation for the range measurement. Our proposed method can measure accurate 3-D coordinates of objects' surfaces in liquid taken for calculating the refraction effect. The effectiveness of the proposed method is shown through experiments. By considering the refraction of the light, the accuracy of the 3-D measurement of objects in water becomes same as that when there is no water, although the accuracy is bad when the refraction of the light is not considered. The accuracy of the 3-D measurement is about 1mm for objects located about 400mm from the laser range finder. The measurement speed can be also reduced as compared with the case of using a spot laser beam.**

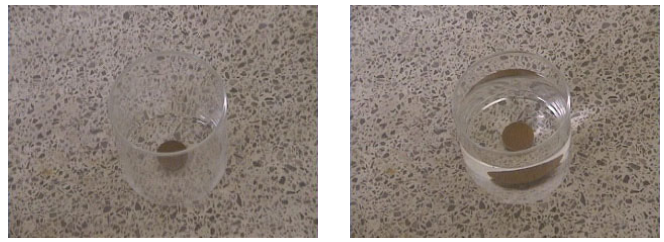
Index Terms*— three-dimensional measurement, laser range finder, objects in liquid, refraction of light, light stripe projection method

I. INTRODUCTION

In this paper, we propose a three-dimensional (3-D) measurement method of objects' shapes in liquid by using the light stripe projection method.

3-D measurement of objects' surfaces is essential technique because there are a lot of applications. Especially, the acquisition of 3-D coordinates of objects' surfaces by vision sensors is preferred because it is a non-contact method and we can get coordinate values of objects in the large range simultaneously.

Robots must work in various spaces now and in the near future, and it is necessary to measure objects not only in the air but also in various environments. Therefore, the 3-D measurement of objects in water is also essential technique. For example, important sample like creatures that are pickled in formalin must be measured with care without contacting. The underwater observation such as the generation of sea floor maps and the examination of the ecology of underwater creatures by underwater robots is also important. However, if cameras and objects are in the different condition where the refraction index differs from each other, several problems occur and a precise measurement cannot be achieved.



(a) Image without water. (b) Image with water.

Fig. 1. Example of image distortion (position change).

Fig. 2. Example of image distortion (size and shape change).

For example, Fig. 1(a) shows an image of a coin inside a cylindrical water tank without water, and Fig. 1(b) shows an image in which the tank is filled with water. In this case, the position of the coin looks different between with and without water, although the real position of the coin never changes.

Similarly, Fig. 2 shows an image of a single rectangular object when water is filled to the middle. In this case, the size and the shape of the object look different between above and below the surface of water.

These problems occur not only when a vision sensor is set outside the liquid but also when it is set inside, because in the latter case we should usually place a protecting glass plate in front of viewing lens or a laser beam scanner.

Therefore, it becomes difficult to measure precise positions and shapes of objects when water exists because of the image distortion by the refraction of the light.

About the 3-D measurement in water, there are several studies. Acoustical methods using sonars are often used[1], [2], especially for underwater robot sensors[3], [4]. These methods can measure the rough shape of the sea floor and detect the existence of a shoal of fish. However, they don't give high resolution due to relatively longer wavelength of ultrasonic waves than that of the light, and are not suitable for 3-D measurement of objects at a short distance with high accuracy. Therefore, the photogrammetric images acquired by cameras are effective for the precise 3-D measurement[5], [6], [7].

For this reason, 3-D measurement methods by using cameras are proposed. However, the methods by using a stereo camera system [7], [8] have the problem that the corresponding points are difficult to detect when the texture of the object's surface is simple in particular when there is the refraction on the boundary between the air and the liquid. The method by the use of motion stereo images obtained with a moving camera[9] also has the problem that the relationship between the camera and the object is difficult to estimate because the camera moves. The surface shape reconstruction method of objects by using an optical flow[10] is not suitable for the accurate measurement, too.

Therefore, we have been proposed a 3-D measurement method of objects in a water tank with a laser range finder[11]. In this method, a spot light from a laser is projected on underwater objects' surfaces from the outside of the water tank for the easy detection of the corresponding points. However, a spot light irradiates only one point on the objects' surfaces, and the wide-range measurement cannot be carried out simultaneously (Fig. 3(a)). It means that it takes a lot of time to measure the whole surface of the object.

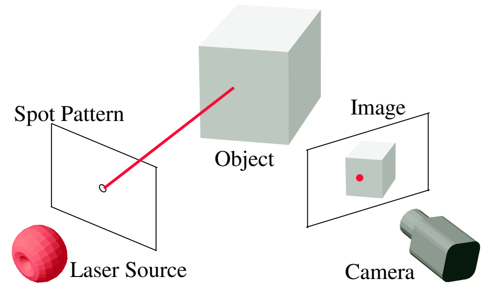
In this paper, we propose a 3-D measurement method of objects in liquid by using a laser range finder with the light stripe projection method that can project a slit light from a laser on the object's surface to realize wide-range measurement (Fig. 3(b)).

The composition of this paper is detailed below. In Section II, the principle of the 3-D measurement is explained. In Section III, the ray tracing method for 3-D measurement is formulated. In Section IV, the process of the 3-D measurement of objects is constructed. In Section V, we verify our method with experiments and Section VI describes conclusions.

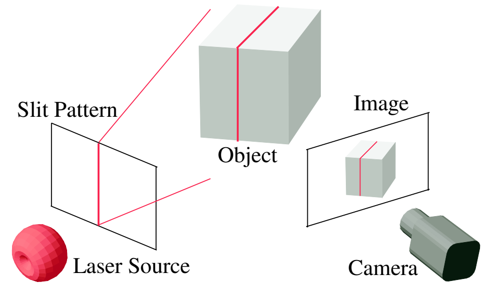
II. Principle of 3-D Measurement

A. Overview of our method

The 3-D coordinates of object surfaces are measured by the camera and the slit laser beam that can change its direction from side to side (Fig. 4). The object is inside a water tank that is filled with the water. In this paper, we use the flat water tank. Of course, our method can treat with an arbitrary shape's water tank in principle, *i.e.* non-flat water tank such as a cylindrical beaker[11].



(a) Spot light.



(b) Slit light.

Fig. 3. 3-D measurement with laser range finders.

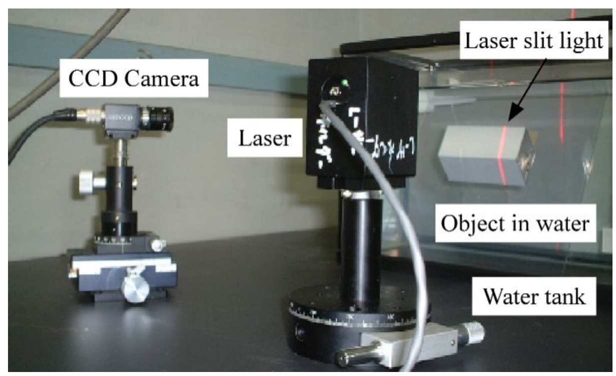


Fig. 4. Overview of 3-D measurement by using the light stripe projection method.

B. Model of 3-D Measurement

The principle of the 3-D measurement is based on a 3-D optical ray tracing technique that properly models the situation. The ray from the camera and that from the laser are traced respectively and the intersection point of two rays corresponds to the surface of the object.

A laser slit light is generated by extending a laser spot light to a line with a spot-to-slit optical system (Fig.5).

It is difficult to detect the point of the object's surface where the ray of the light whose angle is ψ reaches, especially when the light is refracted. This is because irradiated area on the object's surface is not a point but a line. Generally, when the light does not be refracted, the shape of the slit light projected on the object's surface forms a certain shape and its shape from a camera view can be expressed as excellent matrix such as fundamental matrix, homography matrix, and so on. However,

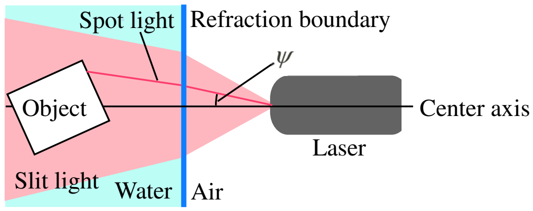


Fig. 5. Laser slit light.

it cannot be expressed in excellent way if the boundary's shape between the air and the water is complicated when the light is refracted.

Therefore, we deal with the slit light as the bunch of the spot light, and the ray tracing of each spot light is executed respectively. It is possible to measure the whole slit light to trace all spot light respectively. This ray tracing method is well known in computer graphics, and is used frequently when the situation is complicated.

C. Definition

Figure 6 shows the model of the optical rays. Here, let the center of the camera lens $O : (0, 0, 0)^T$ be the origin of the world coordinate. Z-axis is set as the same direction of the optical axis of camera, and X-axis and Y-axis are set as the perpendicular direction to Z-axis.

The boundary between the air and the water (the surface of water tank) and the laser slit light are set parallel to Y-axis, respectively. We define "light spread angle ψ" as the angle between the XZ-plane and each laser spot light that is construct the laser slit light (Fig. 5).

In the world coordinate, let $K : (x_g, y_g, z_g)^T$ be the intersection point between the ray from the camera and the boundary of the refraction of the light, $O' : (x_0, y_0, z_0)^T$ be the origin of the laser, $M : (x'_g, y'_g, z'_g)^T$ be the intersection point between the ray from the laser and the boundary of the refraction of the light. We define $Q : (u, v)$ as the position of the spot light in the image plane, and G as the intersection point between Z-axis and the boundary of the refraction of the light.

Additionally, let f be the image distance[1], h be the distance between O (principle point of the camera) and the boundary of the refraction of the light G, ϕ be the angle between X-axis and the boundary of the refraction of the light, θ be the angle between the ray from the laser and the boundary of the refraction of the light in XZ-plane, η be the angle between the laser and X-axis in XZ-plane, i and r be the angle of incident and angle of refraction about the ray from the camera, respectively, and i' and r' be the angle of incident and angle of refraction about the ray from the laser, respectively.

And we define $(\lambda, \mu, \nu)^T$ as the unit normal vector of the boundary of the refraction of the light, $(\alpha, \beta, \gamma)^T$ and $(\alpha_g, \beta_g, \gamma_g)^T$ as the unit vector before and after refraction from the camera, respectively, and $(\alpha', \beta', \gamma')^T$ and $(\alpha'_g, \beta'_g, \gamma'_g)^T$ as the unit vector before and after refraction from the laser, respectively.

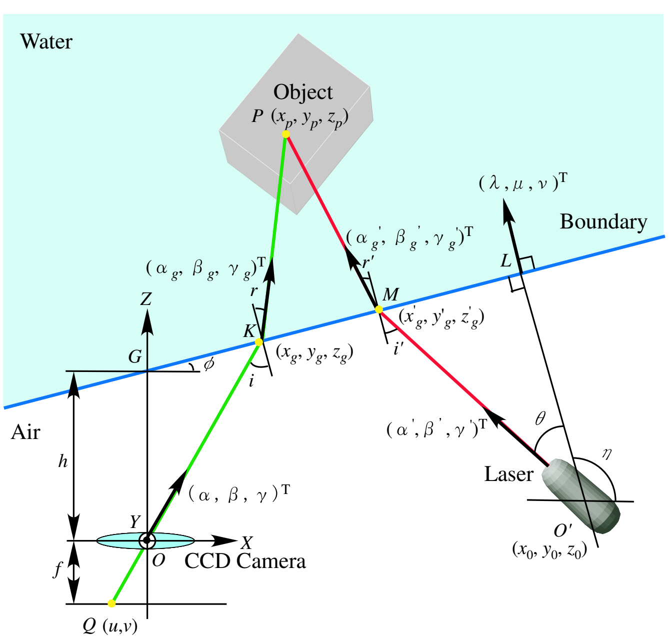


Fig. 6. Principle of 3-D measurement.

III. Ray Tracing for 3-D Measurement

A. Ray Tracing from Camera

In this study, a pinhole camera model is adopted. The coordinate $(u, v)^T$ on the image plane is translated to the coordinate $(x, y, z)^T$ on the world coordinate.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & f \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}, \tag{1}$$

where f is the image distance and a_{ij} are the other camera parameters. The direction vector of the ray from the camera is expressed as:

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{\sqrt{x^2 + y^2 + f^2}} \begin{pmatrix} x \\ y \\ f \end{pmatrix}. \tag{2}$$

On the other hand, the unit normal vector of the refraction boundary $(\lambda, \mu, \nu)^T$ is expressed as follows because the refraction boundary is perpendicular to the XZ-plane.

$$\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} = \begin{pmatrix} -\sin\phi \\ 0 \\ \cos\phi \end{pmatrix}. \tag{3}$$

The intersection point between the ray from the camera and the refraction boundary $K : (x_g, y_g, z_g)^T$ is expressed

[1]The image distance is equal to the distance between the center of lens and the image plane. Although it is confusable, the image distance is not same as the focal length. When an image of an infinitely (or at least sufficiently) distant object is created on the sensor, this distance is equal to the focal length of the lens[12].

as follows by using α, β, γ, ϕ, and h:

$$\begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix} = \frac{h}{\gamma - \alpha \tan\phi} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}. \quad (4)$$

In the next step, the ray from the camera after the refraction of the light must be traced. From the law of the refraction, the unit vector of the ray $(\alpha, \beta, \gamma)^T$, $(\alpha_g, \beta_g, \gamma_g)^T$, and the unit normal vector of the refraction boundary $(\lambda, \mu, \nu)^T$ are on the same plane. Therefore, $(\alpha_g, \beta_g, \gamma_g)^T$ can be expressed as the linear sum of $(\alpha, \beta, \gamma)^T$ and $(\lambda, \mu, \nu)^T$.

$$\begin{pmatrix} \alpha_g \\ \beta_g \\ \gamma_g \end{pmatrix} = p \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} + q \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}, \quad (5)$$

where p and q are the constants.

Because $(\alpha, \beta, \gamma)^T$ and $(\lambda, \mu, \nu)^T$ are unit vectors, the following equations are obtained:

$$\alpha^2 + \beta^2 + \gamma^2 = 1, \quad (6)$$

$$\lambda^2 + \mu^2 + \nu^2 = 1. \quad (7)$$

The inner and outer products of these vectors are calculated as:

$$\cos i = \alpha\lambda + \beta\mu + \gamma\nu, \quad (8)$$

$$\sin^2 i = (\beta\nu - \gamma\mu)^2 + (\gamma\lambda - \alpha\nu)^2 + (\alpha\mu - \beta\lambda)^2. \quad (9)$$

About the unit vector after the refraction of the light $(\alpha_g, \beta_g, \gamma_g)^T$ and the unit normal vector of the refraction boundary $(\lambda, \mu, \nu)^T$, the inner and the outer products can be calculated in the same way.

$$\cos r = \alpha_g\lambda + \beta_g\mu + \gamma_g\nu, \quad (10)$$

$$\sin^2 r = (\beta_g\nu - \gamma_g\mu)^2 + (\gamma_g\lambda - \alpha_g\nu)^2 + (\alpha_g\mu - \beta_g\lambda)^2. \quad (11)$$

When the Snell's low of the refraction is applied, the following equation is gained:

$$\frac{\sin r}{\sin i} = \frac{n_1}{n_2}, \quad (12)$$

where n_1 and n_2 are the refraction index before and after the refraction, respectively (n_1: the refraction index of the air, n_2: that of the water). From (5), and (8)–(12), p and q can be calculated. As the result, the unit vector of the ray after the refraction $(\alpha_g, \beta_g, \gamma_g)^T$ is gained as:

$$\begin{pmatrix} \alpha_g \\ \beta_g \\ \gamma_g \end{pmatrix} = \frac{n_1}{n_2} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} + \left(\cos r - \frac{n_1}{n_2} \cos i \right) \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}. \quad (13)$$

The ray from the camera finally reaches on the surface of the object at the point $P : (x_p, y_p, z_p)^T$:

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = s \begin{pmatrix} \alpha_g \\ \beta_g \\ \gamma_g \end{pmatrix} + \begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix}, \quad (14)$$

where s is a constant.

The 3-D coordinate values $P : (x_p, y_p, z_p)^T$ is obtained when parameter s is gained.

B. Ray Tracing from Laser

The ray tracing from the starting point of the laser can be executed in the same way of the camera.

The intersection point between the ray from the laser and the refraction boundary $M : (x'_g, y'_g, z'_g)^T$ is expressed as:

$$\begin{pmatrix} x'_g \\ y'_g \\ z'_g \end{pmatrix} = \frac{h + x_0 - z_0}{\gamma' - \alpha' \tan\phi} \begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix}, \quad (15)$$

where

$$\begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix} = \begin{pmatrix} \cos\psi \cos\eta \\ \sin\psi \\ \cos\psi \sin\eta \end{pmatrix}. \quad (16)$$

Here, η can be estimated from the calibration of the laser equipment because η is the angle of all spot light (that constructs slit light). However, ψ is not estimated from the calibration because each spot light cannot be distinguished with each other.

Therefore, ψ is regarded as an unknown parameter, and is estimated from the numerical calculation for the last time.

After tracing the ray from laser by considering the refraction of the light, the unit vector of the ray after the refraction $(\alpha'_g, \beta'_g, \gamma'_g)^T$ is obtained as follows:

$$\begin{pmatrix} \alpha'_g \\ \beta'_g \\ \gamma'_g \end{pmatrix} = \frac{n_1}{n_2} \begin{pmatrix} \alpha' \\ \beta' \\ \gamma' \end{pmatrix} + \left(\cos r' - \frac{n_1}{n_2} \cos i' \right) \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}. \quad (17)$$

Finally, the ray from the laser reaches on the surface of the object at the point $P : (x_p, y_p, z_p)^T$.

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = t \begin{pmatrix} \alpha'_g \\ \beta'_g \\ \gamma'_g \end{pmatrix} + \begin{pmatrix} x'_g \\ y'_g \\ z'_g \end{pmatrix}, \quad (18)$$

where t is a constant.

C. Determination of 3-D Coordinate Value

The point on the surface of the object coincides both the result of the ray tracing from the camera and that from the laser.

The unknown parameters in (14) and (18) are s, t, and ψ. Therefore, three parameters can be calculated by considering (14) and (18) as the simultaneous equations.

$$s\alpha_g + x_g = t\alpha'_g + x'_g, \quad (19)$$
$$s\beta_g + x_g = t\beta'_g + x'_g, \quad (20)$$
$$s\gamma_g + x_g = t\gamma'_g + x'_g. \quad (21)$$

From (19)–(21), a polynomial equation of the trigonometric function about ψ is gained. We solve it by numerical calculation because the polynomial equation of trigonometric function cannot be solved analytically.

In concrete terms, parameters s and t are obtained as the function of ψ from (19) and (21) at first. Then, an equation whose unknown parameter is only ψ can be gained by substituting obtained s and t for (20):

$$\begin{aligned} & n_1 \sin\psi\big(\alpha_g(a_3a_4 + a_5a_6) - a_5(\alpha_g a_6 \gamma_g a_4)\big) \\ & -n_2 a_4\big(\beta_g(a_3a_4 + a_5a_6) + y_g(\alpha_g a_6 + \gamma_g a_4) \\ & \quad -(\alpha_g a_6 - \gamma_g a_4)(a_2 \sin\psi + y_0)\big) = 0, \quad (22) \end{aligned}$$

where

$$a_1 = \frac{n_1}{n_2}\cos\psi\sin(\eta-\phi) - \sqrt{1 - \frac{n_1^2}{n_2^2}\left(1 - \cos^2\psi\sin^2(\eta-\phi)\right)}, \quad (23)$$
$$a_2 = \frac{z_0 - h - x_0\tan\phi}{\alpha'\tan\phi - \gamma'}, \quad (24)$$
$$a_3 = z_g - a_2\cos\psi\sin\eta - z_0, \quad (25)$$
$$a_4 = \frac{n_1}{n_2}\cos\psi\cos\eta + a_1\sin\phi, \quad (26)$$
$$a_5 = a_2\cos\psi\cos\eta + x_0 - x_g, \quad (27)$$
$$a_6 = \frac{n_1}{n_2}\cos\psi\sin\eta - a_1\cos\phi. \quad (28)$$

The unknown parameter ψ is calculated when (22) is satisfied. Conclusively, the surface of the object $P : (x_p, y_p, z_p)^T$ is obtained.

IV. Procedure of 3-D Measurement

A. Calibration

We must calibrate the camera parameters, the starting point and the direction vector of the laser beam ($O' : (x_0, y_0, z_0)^T$ and η), the distance between the principle point of the camera and the boundary of the refraction (h), and the angle between XY-plane and the boundary of the refraction (ϕ).

At first, the camera parameters are calibrated by using the planner pattern on which surface checked patterns are drawn.

Next, h and ϕ are calibrated. These unknown parameters can be also estimated by using the planner pattern stuck on the boundary of the refraction. They are estimated from the distance between checked patterns by using the least squares method.

Finally, the starting point O' and the direction of the laser beam η are calibrated by changing the direction of the laser. A spot laser is used when O' and η are estimated because of the uniqueness of the irradiated point.

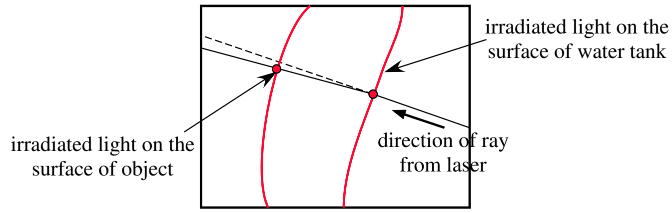


Fig. 7. Extraction of laser point.

In this way, the relationship of the camera, the laser, and the water tank is calibrated.

B. Extraction of Laser on Object

About the extraction of the spot light (that constructs the slit light) of the laser in the acquired images, the epipolar constraints and the subpixel calculation are utilized.

Rough areas of the laser beam in the images are limited by the epipolar constraints. When multiple points are detected as the laser light, we can judge the point on the surface of the object that the objective point exists in the deepest direction of the epipolar line in any cases (Fig. 7).

The subpixel measurement is executed by calculating the center of gravity of the extracted pixels that belong to the laser.

C. 3-D Shape Measurement

After all the calibration and the extraction of laser in the image, 3-D shape measurement of objects in liquid is executed. A cross-sectional surface of the object can be gained from one image, and another cross-sectional shape can be acquired while the direction of the laser is changing. When the laser finishes the change of the direction, whole shape of the object is measured.

V. Experiments

A slit light is generated by a semiconductor laser machine whose wavelength is 633nm. The refraction index of the air is set as $n_1 = 1.000$ and the water as $n_2 = 1.335$. The resolution of images is 640 $\times$ 480pixel. The camera parameter f is estimated as 1079.0pixel from the camera calibration.

3-D measurement of an rectangular object whose length of one edge is 40.3mm and that in the water tank filled with water is executed.

Figure 8 shows an example of experimental results when two faces of the rectangular object can be seen from the camera. There is no outlier of measured point and proper coordinate values can be gained.

To evaluate the proposed method quantitatively, the accuracy of the shape reconstruction is compared between when the water tank is filled with the water and when the tank is not filled with water.

The accuracy of the shape reconstruction from the measured 3-D coordinates of object's surface is evaluated as follows. At first, irradiated points by the laser light in the image is extracted, and center of gravity is calculated for each scanning

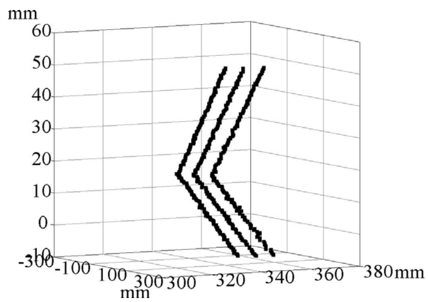
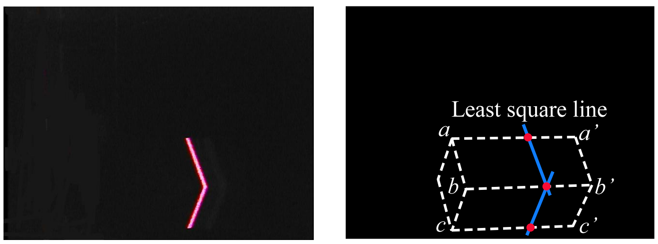


Fig. 8. Experimental result I.



(a) Laser image. (b) Extracted lines.

Fig. 9. Shape reconstruction.

line (Fig.9(a)). Next, lines by the least square estimation from the measured positions are searched, and 3-D coordinates of endpoints of these lines are calculated (Fig.9(b)). In the same way, 3-D coordinates of endpoints are calculated while η changes. From these coordinate values, 3-D shape of the object is reconstructed by the least square estimation under the constraints that each edge is parallel with each other in the case of rectangular objects. Finally, the distance between two edges are calculated and compared with the real length.

The experiments were done while changing the distance between the laser range finder and the water tank.

Table I indicates the average error, the maximum error, and the standard deviation between the real shape and reconstructed shape of the object located not over 400mm from the laser range finder. The accuracy with water is almost the same as the accuracy without water, although the accuracy with water is bad when the refraction of the light is not considered. The maximum error rate is 2.7%.

These results show that our method can work without failure regardless of the existence of water by considering the refraction of the light.

VI. Conclusions

In this paper, we propose a 3-D measurement method of objects in liquid with a laser range finder by using the light stripe projection method. We take the refraction of the light into consideration in the triangulation, and build the technique that is suitable for objects in liquid. Experiments have verified the effectiveness of proposed 3-D measurement method. By considering the refraction of the light, the accuracy of the 3-D measurement of objects in water becomes same as that when there is no water, although the accuracy is bad when the refraction of the light is not considered. The accuracy of the results is within subpixel order of acquired images. The measurement speed can be also reduced as compared with the case of using a spot laser.

As the future works, it is desirable that various shapes of objects are measured. The method by using a sequence of 2-D structured (encoded) illuminations can cover a wide range and objects can be measured quickly. Estimation of refraction index of each region is also needed. In the last result, simultaneous measurement of the shape of water tank and the object inside the tank must be considered.

TABLE I

Accuracy (Error) of 3-D Measurement.

Situation	Average	Maximum	Standard deviation
Without water	0.7mm	1.0mm	0.36mm
With water	0.7mm	1.1mm	0.46mm

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